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Order of Operations

When parentheses are used in a problem involving two or more operations, they indicate the order in which the operations are to be performed. The operation(s) within the parentheses must be performed first – from the inside out; then, perform the remaining operations according to the order of operations covered below.

For example: $(3.4 \times (4.0 + 6.1))/2 = ?$

$$(3.4 \times \underline{10.1})/2 = 17.17$$

P.E.M.D.A.S	
Is the acronym for the order of operations	
1. Parentheses	Calculate in order from the innermost to the outermost, completing the operations within the parentheses following steps 2-6 below.
2. Exponents	$(4^2 = 4 \times 4)$ $(4^3 = 4 \times 4 \times 4)$
3. Multiplication	2×3 , or $2(3)$, or $2 \cdot 3$
4. Division	$4 \div 2$, or $4/2$, or $2 \overline{)4}$
5. Addition	
6. Subtraction	

Ratio & Fractions

A Ratio is a numerical comparison between 2 quantities -- how much of one thing compared to another thing. Ratios can be expressed as fractions, decimals or percentages (%). We use ratios when mixing proportions of 2 elements or ingredients. Once we know the required or desired ratio for the 2 elements we can scale the mixture up or down to meet the needs of the job.

For example:

- 3 gallons of water to 2 cups of lemonade crystals = $3/2$ or 1.5
- 5 bags cement to 10 gallons water = $5/10$ or .5 or 50%
- 9 parts fine aggregate to 20 parts coarse aggregate = $9/20$ or .45 or 45%

The order in which you compare the two quantities is important!

For example: Your ideal concrete mix has 67.5lbs of water and 150lbs of cement

- What is the cement to water ratio?

$$150\text{lbs} / 67.5\text{lbs} = \underline{\underline{2.222}}$$

This means that for every pound of cement you will need to add 2.222 (or a little more than twice as much in weight) the number of pounds of water.

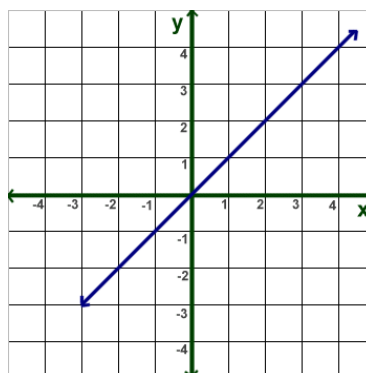
- What is the water to cement ratio?

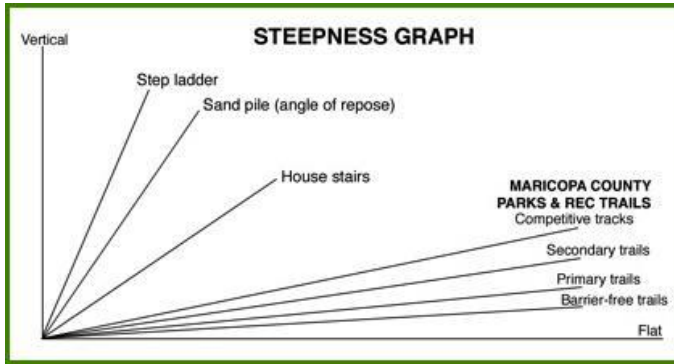
$$67.5\text{lbs}/150\text{lbs} = \underline{\underline{.45}}$$

This means that for every pound of water you will need to add .45 times (a little less than $\frac{1}{2}$ as much in weight) the number pounds of cement.

A slope, or rise over run, is also expressed as a ratio; a comparison of height and distance (steepness).

Thinking about slope as a ratio, what does a slope of 1 tell you? It would mean that your vertical and horizontal distances are identical, because $1 = 5/5 = 2/2 = 10/10$, etc. A slope of 1 is represented on the graph below.





Up which slope in this graph would you prefer to hand-carry a load of rocks?

A Fraction expresses a number that is part of a whole. Although ratios can look like fractions, a ratio is a comparison (of part to part) while a fraction expresses a number (that is part of a whole).

We know that the fraction $\frac{1}{2}$ is a *set numerical quantity* that we can measure in space. We can express the same fraction of $\frac{1}{2}$ inch in different ways ($\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{50}{100}$), but we would have the same *numerical* measurement, right? That quantity of $\frac{1}{2}$ inch as we measure it on a ruler does not change.

However, a ratio of $\frac{1}{2}$ (or 1:2) represents a *relationship* between two quantities. Those quantities can change, but the relationship must stay the same. In ratios, the relationship is important. It is, “How much of this versus that?”

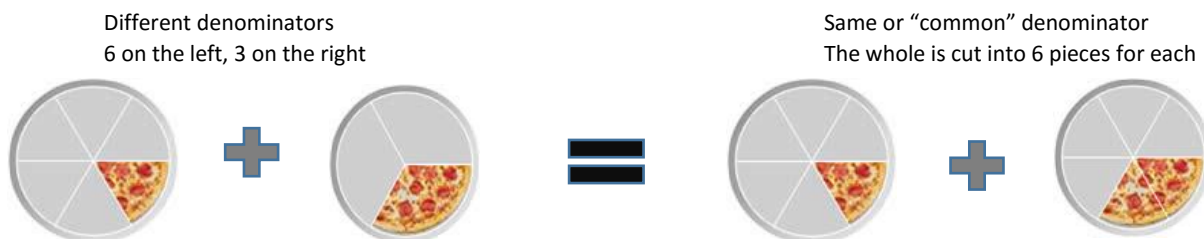
The top number of a fraction is called the numerator.

The bottom number of a fraction is the denominator.

ADDING and SUBTRACTING Fractions

When you are adding or subtracting fractions, the denominators must be the same. This is referred to as a “common denominator.”

For example: $\frac{1}{6} + \frac{1}{3}$ (illustrated in the pictures below).



Before we can add these two fractions, we must first determine the least or lowest common denominator. In other words, the “whole” from which the fraction is derived must be cut into the same number of slices. The fractions on each side of the equal sign represent the same quantities. However, the fractions on the right have a common denominator of 6: $\frac{1}{6} + \frac{2}{6}$.

To find the least common denominator, **list the multiples** of each denominator (multiply by 2, 3, 4, etc. out to about 6 or seven usually works) then look for the **smallest number** that appears in both lists.

Example: So to add $1/3 + 1/6$ we would find the least common denominator as follows:

1. First we list the **multiples** of each denominator.
 Multiples of **3** are 3, 6, 9, **12**, 18, 24, ...
 Multiples of **6** are 6, **12**, 18, 24, 30, 36, 42, 48, ...
2. When you look at the list of multiples, you can see that 12 is the smallest number that appears in both lists.
3. The **least common denominator** of $1/3$ and $1/6$ is **12**.

We would next rewrite the equivalent fraction for $1/3$ and $1/6$, with 12 as the denominator for each. To do so, we would

1. Divide the least common denominator by the denominator of the fraction.
2. Multiply the answer times the numerator of the fraction.
3. Re-write the fraction using the least common denominator as the denominator and the answer from step 2 as the numerator.

Back to our example: $1/3 + 1/6$, LCD = **12**

	Re-writing $1/3$ with LCD	Re-writing $1/6$ with LCD
Step 1	$12 \div 3 = 4$	$12 \div 6 = 2$
Step 2	$4 \times 1 = 4$	$2 \times 1 = 2$
Step 3	$4/12$	$2/12$

Now that we have converted these to fractions with common denominators, we can add or subtract:

$$4/12 + 2/12 = 6/12 = 1/2 \quad \text{or} \quad 4/12 - 2/12 = 2/12 = 1/6$$

Multiplying and Dividing Fractions

We don't have to worry about common denominators when multiplying or dividing fractions! To multiply fractions, we

	$2/3 \times 3/5 = ?$	
Step 1	Multiply numerators with numerators & denominators with denominators	$2 \times 3 = 6$ $3 \times 5 = 15$
Step 2	Reduce fraction if possible	$6/15 = 2/5$

To divide fractions, we

$2/3 \div 3/5 = ?$		
Step 1	Invert or “flip” the divisor (2 nd fraction) & change the division sign to a multiplication sign	$2/3 \div 3/5$ becomes $2/3 \times 5/3$
Step 2	Complete the multiplication of the numerators and denominators	$2 \times 5 = 10$ $3 \times 3 = 9$
Step 3	Reduce fraction if possible	$10/9 = 1 \frac{1}{9}$

CONVERSIONS

In order to compute most math operations, the units of measure must be the same. For example, we add lengths together (inches plus feet plus yards, etc.) and we can add area quantities together (100 sq.ft. plus 10 sq. yds). But we cannot add a quantity of 15 inches (linear measure) to a quantity of 100 sq.ft. (area). Think about this. If you measure the area of a 10 ft x 10 ft room to be 100 sq.ft. and someone asked you to “add 15 inches” to that total, that request would not make sense.

We can convert the units within each geometric operation (linear, area, volume, etc.) to smaller or larger scale by using conversion factors. If you need to convert a measurement from a larger to a smaller unit, you multiply the measure by the conversion factor. To convert up to a larger unit of measure, you would divide by the conversion factor as demonstrated below where the conversion factor is 12.



12 inches = 1 foot

3 feet = **36 Inches**

inches \div 12 to convert inches to feet

48 inches = **4 Feet**

feet \times 12 to convert feet to inches

Other common conversion factors are provided on the next page.

<u>Multiply</u>	<u>Times</u>	<u>To Find</u>
Cubic Yards	46,656	Cubic Inches
Cubic Yards	202.0	Gallons
Grams	0.03527	Ounces
Ounces	28.35	Grams
Ounces	0.0625	Pounds
Pounds	453.6	Grams
Tons	2000	Pounds
Pounds	0.0005	Tons
Gallons	8.3	Pounds of Water
Gallons	0.1337	Cubic Feet
Gallons	231	Cubic Inches
Pounds of Water	0.1198	Gallons
Miles per Hour	88	Feet Per Minute
Miles Per Hour	1.467	Feet Per Second

$$\text{Slope Ratio} = \frac{\text{Vertical}}{\text{Horizontal}}$$

$$\text{Percent grade} = \frac{\text{Vertical rise or fall}}{\text{Horizontal Distance}} \times 100$$

1 gallon of water weighs 8.3 pounds

<u>Multiply</u>	<u>Times</u>	<u>To Find</u>
Feet	12	Inches
Feet	0.3333	Yards
Miles	5280	Feet
Miles	1760	Yards
Square Feet	0.1111	Square Yards
Acre	43,560	Square Feet
Acre	4840	Square Yards
Square Miles	640	Acres
Square Yards	9	Square Feet
Cubic Feet	62.4	Pounds of Water
Cubic Feet	1728	Cubic Inches
Cubic Feet	0.03704	Cubic Yards
Cubic Feet	7.48	Gallons
Cubic Inches	.0005787	Cubic Feet
Cubic Inches	.00002143	Cubic Yard
Cubic Yards	27	Cubic Feet

$$\text{Percent moisture (based on dry weight)} = \frac{\text{Wet weight} - \text{dry weight}}{\text{Dry weight}} \times 100$$

To convert fractions into decimals, simply divide the numerator by the denominator. For example, to convert 1/2 into a decimal calculate $1 \div 2 = .50$, $2/5 = 2 \div 5 = .40$, and so on.

WORD PROBLEMS

A word problem is a situation when you need to solve a mathematical operation and provide a numerical solution when much of the background information you are given is in text form. This happens often in everyday life and on the job. Rarely do individuals write down equations and ask you to solve for X .

For Example: Your supervisor gives you 525 boxes of bolts. Your job is to equally distribute them to the five job sites. How many boxes does each site get?

In the format of a mathematical equation, this problem looks like this: **525 boxes/5 = 105**. Our supervisor will ask us to make this calculation, but will give us the information to do so in sentence or verbal form. We have to determine how to translate that “problem” into a mathematical equation so that we can calculate an answer. The following steps will help guide you through the translation process.

SOLVING WORD PROBLEMS	
Step 1	Read the problem through entirely & draw diagrams (if possible or helpful)
Step 2	List the variables, the “things” or “stuff” they are talking about that can be measured or quantified
Step 3	Write down the <i>unknown</i> , what do they want to know? What unit of measure?
Step 4	Estimate the answer using your real-world experience (“Guesstimate”)
Step 5	Identify “key” words that suggest particular math operations (see table below)
Step 6	Write the equation (or equations) that mirrors problem

Key words for Mathematical Operations	
Addition	Increased by, more than, combine, total of, sum, added to, plus
Subtraction	Difference, decreased by, less than, fewer than, take away, how many left
Multiplication	Product of, times, of, factor of, double, triple
Division	Quotient, divided by, ratio of, per, out of, percent
Equals	Is/are, was/were, will be, gives, yields, sold for, would you have

Let’s work through a word problem using these steps on the next page.

1. A tank truck weighs 5 tons empty. Loaded, it weighs 14.3 tons. How many gallons of water is it carrying? One gallon of water weighs 8.3 pounds.

Step 2: *List the variables or quantities provided*

Truck empty = 5 tons Truck full = 14.3 tons 1 gal. water = 8.3 lbs

Step 3: *The unknown*, or what we need to find out is how much water the full truck is carrying.

Step 4: Can you “*guesstimate*” this one? This one could be difficult to estimate because of the conversion factors from tons of water to gallons of water. But if you work with tons of water often you might be able to provide an estimate.

Step 5: *Key words* – although we don’t see the exact key words from the table, we know that we have a weight for the truck when it is full and when it is empty. That describes a “difference” in this truck’s weight before and after picking up the load. When a difference is indicated, that tells us that we have a subtraction to perform – subtracting the empty weight from the full weight to determine the weight of the load.

We are asked to find the quantity of water in that load. If we complete the subtraction operation we just mentioned, we will know how much water there is in tons. But we are asked how many gallons of water is the truck carrying, not tons.

We need to *convert* the #tons of water into #gallons of water. This tells us we will have a conversion to calculate. Let’s put these operations into mathematical form.

Step 6: *Write the equation (or equations) that mirrors problem*

$$14.3 \text{ tons} - 5 \text{ tons} = 9.3 \text{ tons} \quad (\text{full truck} - \text{empty truck} = \text{load weight})$$

$$9.3 \text{ tons} \times 2000 \text{ lbs} = 18,600 \text{ lbs of water} \quad (\text{conversion, } 1 \text{ ton} = 2000 \text{ lbs})$$

$$18,600 \div 8.3 = \underline{\underline{2240.96 \text{ gallons of water}}} \quad (\text{conversion, } 1 \text{ gal water} = 8.3 \text{ lbs})$$

ADVANCED CALCULATIONS

Average

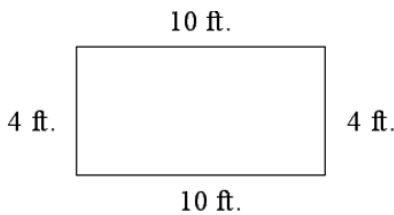
$$\text{Average} = \frac{\text{The sum of all the numbers you're averaging}}{\text{The number of numbers you're averaging}}$$

For Example: The widths of seeding pad in several different locations are 25, 30, 35, 35, 44, 46, and 32 feet. What is the average width of these pads?

$$\frac{25 + 30 + 35 + 35 + 44 + 46 + 32}{7} = \frac{247}{7} = \underline{\underline{35.29 \text{ feet}}}$$

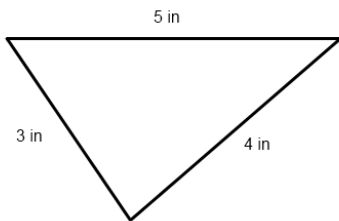
Perimeter is the linear distance around any flat object. We measure the outside edge of the object.

For example: Calculate the perimeter of this rectangle.



$$10 \text{ ft} + 4 \text{ ft} + 10 \text{ ft} + 4 \text{ ft} = \underline{\underline{28 \text{ feet}}}$$

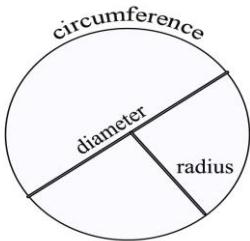
Now calculate the perimeter of the triangle below.



$$5 \text{ in.} + 4 \text{ in.} + 3 \text{ in.} = \underline{\underline{12 \text{ inches}}}$$

Calculating the perimeter (called the circumference) of a circle is a bit different, because we are not measuring straight lines. The formula for calculating the circumference of a circle is

2 x π x radius of the circle OR **2πr**, where π = 3.14 (π can be located on most calculators as well)



The parts of the circle you will need to know as we continue working with circular shapes are:

Radius (r) – a line from the center of the circle to the outside edge

Diameter (d) – a line from edge to edge of the circle, passing through the center. You can see that the diameter is also twice the length of the radius, or $d = 2r$

For example: A manhole has a diameter of 48 inches. What is the radius? What is the circumference?

$$d = 2r \text{ or } r = \frac{1}{2}d \quad r = \frac{1}{2}(48) = \underline{\underline{24 \text{ inches}}}$$

Step 1. $\pi \times 48$

Step 2. $3.14 \times 48\text{in} = \underline{\underline{150.72 \text{ inches}}}$

Area is the amount of space inside the boundaries of a 2-dimensional (flat) object.

Area of **square** or **rectangle** = length x width or $l \times w$

Area of a **triangle** = $\frac{1}{2}$ base x height or $\frac{1}{2} bh$

Area of a **circle** = π x radius x radius or πr^2

Area of a **trapezoid** is $\frac{1}{2}$ the sum of the 2 bases x height $\frac{1}{2} (b_1+b_2) \times h$



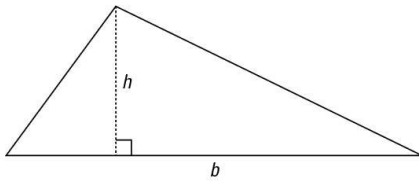
Each side of this box measures 4 feet
Area of this box is 4ft x 4ft = 16 ft²



3ft

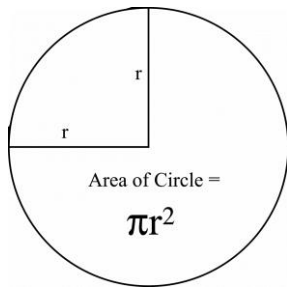
Area of this rectangle is 4 ft x 3 ft = 12 ft²

4 ft



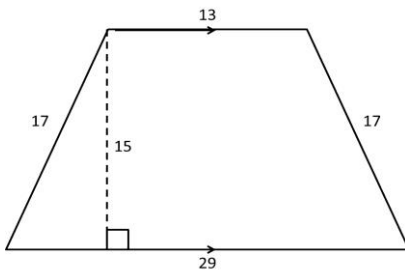
$h = 3\text{ft}$ $b = 7\text{ft}$

Area of this triangle is 3 ft x 7 ft = 21 ft²



$R = 11\text{ ft}$

Area of this circle is $\pi \times (11)^2 =$ 380.13 ft²



$A = \frac{1}{2} (b_1+b_2) \times h$

$A = \frac{1}{2} (29\text{ft} + 13\text{ft}) \times 15\text{ft} = 315\text{ft}^2$

$A = \frac{1}{2} (42\text{ft}) \times 15\text{ft}$

$A = 21\text{ft} \times 15\text{ft} =$ 315ft²

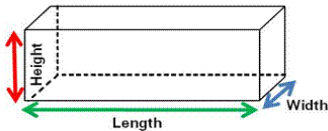
Volume is the amount of 3-dimensional space an object occupies, or the amount of space inside an object like a drum, a ball, a box, etc.

One way to remember the equation for volume is to understand that volume equals the area of the shape times its height.

Volume of **box** = length x width x height or $l \times w \times h$

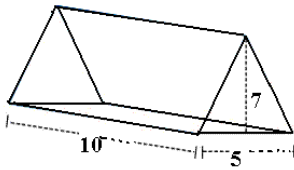
Volume of a **prism (3D triangle)** = $\frac{1}{2}$ base x height x length or $\frac{1}{2} bh \times l$

Volume of a **cylinder** = π x radius x radius x height or $\pi r^2 h$

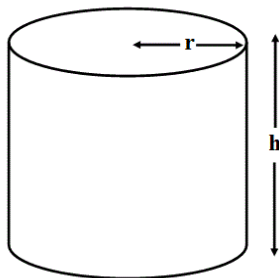


$$w = 3 \text{ ft} \quad l = 7 \text{ ft} \quad h = 2.5 \text{ ft}$$

$$\text{Volume of this box is } 3\text{ft} \times 7\text{ft} \times 2.5\text{ft} = \underline{52.5 \text{ ft}^3} \text{ (cubic feet)}$$



$$\text{Volume of this triangular prism is } \frac{1}{2}(5 \text{ ft} \times 7 \text{ ft} \times 10 \text{ ft}) = \underline{175 \text{ ft}^3}$$



$$R = 4.5 \text{ ft} \quad h = 11 \text{ ft}$$

$$\text{Volume of this cylinder is } \pi (4.5\text{ft} \times 11\text{ft}) = \underline{155.50 \text{ ft}^3}$$

Call or email Terri Nihil @ Nebraska LTAP if you have questions or would like more practice with these mathematical concepts.

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